Adaptive Local Cosine transform for Seismic Image Compression

Aparna.P [1], Dr Sumam David [2]
[1]Research Scholar, Dept.of E&C, NITK, Surathkal, aparnadinesh@rediffmail.com
[2]Senior IEEE Member, Dept. of E&C, NITK, Surathkal, sumam@ieee.org

Abstract

A typical seismic analysis involves collection of data by an array of seismometers, transmission over a narrow-band channel, and storage of data for analysis. Transmission and archiving of large volumes of data involves great cost. Hence there is a need to devise suitable methods for compressing the seismic data without compromising on the quality of the reconstructed signal. This paper presents our work on the seismic data compression based on adaptive local cosine transform and its associated multi-resolution and best-basis methodology and compares the results with wavelet based implementation.

Key Words: Local cosine transform, orthonormal.

1. Introduction

The basic idea behind transform coding of images is that the coefficients of the transformed image are nearly uncorrelated and have an energy distribution more suitable for coding, than the pixels in the spatial domain. The optimal transform for image coding in terms of mean square error is the Karhunen-Loeve Transform (KLT), but since it is signal dependent and computationally complex the KLT is not used in practical applications. Among the various sub-optimal transforms the Discrete Cosine transform (DCT) is most widely used as it is signal independent. It is close to the KLT in terms of energy compaction and rate distortion and has fast network implementation algorithms. Despite of all its advantages the DCT based algorithm at low bit rates, exhibits the blocking effect. The blocking effect is a natural consequence of the independent processing of each block and its incapability to adapt to the patterns in the image. It is perceived in images as visible discontinuities across block boundaries. Seismic data are quite different from the typical images used in multimedia applications. It has a very vast data dynamic range along with significant amount of coherent noise. Also it is a non-stationary signal with extensive oscillatory nature. Hence Local Cosine Transform (LCT) is adopted as a new method for the reduction and smoothing of the blocking effect that appears in DCT based image-coding algorithms at low bit rates[4].The principle of overlapping between adjacent blocks is used with the existing DCT- based encoders by applying a preprocessing phase on the source image.The idea is to use smooth cut-off functions to split the signal and to fold the overlapping parts back into pieces such that the orthogonality is preserved. The folded signal is suited for representation by a trigonometric basis. Such a basis is called smooth local trigonometric basis. These basis functions are nothing but the trigonometric functions multiplied by smooth bell-shaped functions.

The implementation is divided into the following sequence: In the beginning we select the local trigonometric transform (or basis), which is best adapted to encode the image. Then the best-basis methodology is applied on the tree of local trigonometric expansions. Finally, the coefficients are quantized using uniform quantization and the sequences of 0s and 1s generated by uniform quantization are entropy coded using a run-length technique and Huffman coding scheme. However this paper does not emphasize on quantization and coding technique.

2. Adaptive Local Trigonometric bases

In order to analyze the local frequency content of the image, we first cut the support of the image into adjacent blocks. Then a local Fourier analysis is performed inside each block. To obtain a better frequency localization, we do not cut the signal abruptly but we use a smooth window function to localize the segment of interest. Local cosine/sine bases constructed by Coifman and Meyer[5] consist of cosines/sines multiplied by smooth, compactly supported bell functions. These localized cosine/sine functions remain orthogonal. The basis element can be characterized by position α, interval I and frequency index k as follows:

$$\psi_{\alpha l k}(x) = \sqrt{\frac{2}{|I|}} b_I(x) Cos[\pi(k+\frac{1}{2})\frac{x-\alpha}{|I|}] \tag{1}$$

$$\psi_{\alpha l k}(x) = \sqrt{\frac{2}{|I|}} b_I(x) Sin[\pi(k+\frac{1}{2})\frac{x-\alpha}{|I|}] \tag{2}$$

Where $b_I(x)$ is a bell function which is a smooth function compactly supported in the interval $[\alpha-\varepsilon, \beta+\varepsilon']$ for $(\alpha+\varepsilon < \beta-\varepsilon')$.This interval contains $I = [\alpha, \beta]$ which is called its nominal support. We can control the window width by I, the left endpoint of the nominal support by the position α and the frequency by the index k.

The properties of the bell function given by,

$$\begin{cases} b_I(x)^2 + b_I(2\alpha - x)^2 = 1, & x \in [\alpha-\varepsilon, \alpha+\varepsilon]; \\ b_I(x)^2 + b_I(2\beta - x)^2 = 1, & x \in [\beta-\varepsilon', \beta+\varepsilon']; \\ b_I(x) = 1, & x \in [\alpha+\varepsilon, \beta-\varepsilon']. \end{cases} \tag{3}$$

ensure the orthonormality of the bases and afford a fast algorithm.

Suppose a sequence α_j is selected to satisfy $(\alpha_j < \alpha_{j+1})\,; lt_{j\to\pm\infty}\{\alpha_j\} = \pm\infty$ and there also exists an accompanying sequence $\{\varepsilon_j\}$ such that $(\alpha_j + \varepsilon_j \le \alpha_{j+1} + \varepsilon_{j+1})$ for all $j \in Z$. Then the functions

$$\psi_{\alpha lk}(x) = \sqrt{\frac{2}{\alpha_{j+1}-\alpha_j}}\, b_{[\alpha_j,\alpha_{j+1}]}(x) Cos[\pi(k+\frac{1}{2})\frac{x-\alpha_j}{\alpha_{j+1}-\alpha_j}] \tag{4}$$

$(j \in Z, k = 0, 1, 2, ...)$ with positive polarity at α_j and negative polarity at α_{j+1} or

$$\psi_{\alpha lk}(x) = \sqrt{\frac{2}{\alpha_{j+1}-\alpha_j}}\, b_{[\alpha_j,\alpha_{j+1}]}(x) Sin[\pi(k+\frac{1}{2})\frac{x-\alpha_j}{\alpha_{j+1}-\alpha_j}] \tag{5}$$

$(j \in Z, k = 0, 1, 2, ...)$ with negative polarity at α_j and positive polarity at α_{j+1}, form an orthonormal basis for the space $L^2(R)$. A two-dimensional local cosine/sine basis can be generated by the tensor products $\psi_{\alpha_x I_x k_x}(x)\psi_{\alpha_y I_y k_y}(y)$ and their nominal supports are the Cartesian product rectangles of the nominal supports of the x and y factors.

3. Bell Functions

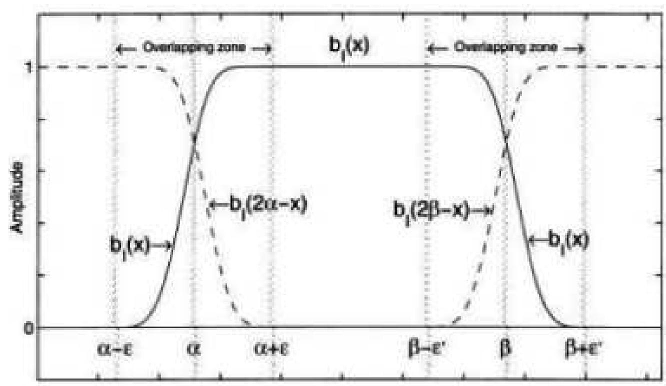

Fig. 1: The bell function $b_I(x)$ [3]

Let $\beta(x)$ be a continuous function defined on R with the following properties:

$$\begin{array}{ll} \text{(i):}\beta(x) = 0 & \text{if } x \le -1; \\ \text{(ii):}\beta(x) = 1, & \text{if } x \ge 1; \\ \text{(iii):}\beta(x)^2 + \beta(-x)^2 = 1, & \text{for all } x. \end{array} \tag{6}$$

In particular, we can use the function $\beta(x)$ defined as following :

$$\beta(x) = \begin{cases} 0 & \text{if } x < -1; \\ Sin\pi/4(1 + Sin\pi/2x) & \text{if } -1 \le x \le 1; \\ 1 & \text{if } x > 1. \end{cases} \tag{7}$$

Hence we define the bell function $b_I(x)$ as follows:

$$b_I(x) = \begin{cases} \beta(x-\alpha/\varepsilon) & x \in [\alpha-\varepsilon, \alpha+\varepsilon]; \\ \beta(\beta-x/\varepsilon') & x \in [\beta-\varepsilon', \beta+\varepsilon']; \end{cases} \tag{8}$$

By this definition the bell has two parts. The left part is an increasing function from 0 to 1 and the right part is a decreasing function from 1 to 0. At the intersection point between these parts the bell has the value 1 as shown in Fig 1.

4. Folding Operation

Rather than calculating inner products with the sequences $\psi_{\alpha Ik}$, we can preprocess data so that the standard fast discrete cosine transform of the fourth type (DCT-IV) or discrete sine transform of the fourth type (DST-IV) algorithm may be used. This can be realized by folding the overlapping parts of the bell functions back into the intervals. Suppose we wish to fold a signal $f(x)$ back into the interval $I = [\alpha, \beta]$ Using the bell function $b_I(x)$ defined by Eq.(8), we have

$f_{new}(x) =$

$$\begin{cases} f_+(x) = b_I(x)f(x) + b_I(2\alpha-x)f(2\alpha-x), \text{ if } \alpha \le x \le \alpha+\varepsilon; \\ f_-(x) = b_I(x)f(x) + b_I(2\beta-x)f(2\beta-x), \text{ if } \beta-\varepsilon' \le x \le \beta; \\ f(x), \text{ if } \alpha+\varepsilon \le x \le \beta-\varepsilon'. \end{cases} \tag{9}$$

The resultant folded data $f_{new}(x)$ is now defined in the interval $[\alpha, \beta]$. To reconstruct $f(x)$ from $f_{new}(x)$, we can use the following unfolding formula:

$f(x) =$

$$\begin{cases} b_I(x)f_{new}(x) - b_I(2\alpha-x)f_{new}(2\alpha-x), \text{ if } \alpha \le x \le \alpha+\varepsilon; \\ b_I(x)f_{new}(x) + b_I(2\beta-x)f_{new}(2\beta-x), \text{ if} \beta-\varepsilon' \le x \le \beta; \\ f_{new}, \text{ if } \alpha+\varepsilon \le x \le \beta-\varepsilon. \end{cases} \tag{10}$$

Edge extension: When the bell shifts to the leftmost endpoint or the rightmost endpoint of the signal, we cannot directly obtain $f_+(x)$ or $f_-(x)$ from the above formula because of the lack of data in the leftmost and rightmost overlapping zones. Hence we go for zero-extension. After the procedures of folding and edge extension, we can apply the fast DCT-IV/DST-IV to the folded data $f_{new}(x)$ to obtain the local cosine/sine transform coefficients.

5. Entropy based Best Basis selection

After we have completed local cosine/sine transforms for all the preset decomposition levels and also calculated the entropy of each transformed sub signal, we can use the Coifman and Wickerhauser fast algorithm[[5]] to search for the local cosine/sine best-basis.

Adapted waveform analysis uses a library of orthonormal bases and efficiency functional to match a basis to give a signal or family of signals. It permits efficient compression of a variety of signals such as speech and images. The pre-defined libraries of modulated waveforms include orthogonal wavelet packets and localized trigonometric functions, have reasonably well controlled time-frequency localization properties. The idea is to build out of library functions an orthonormal basis relative to which the given signal or collection of signals has the lowest information cost. Several cost functions are useful; one of the most attractive is Shannon entropy, which has a geometric interpretation in this context.

1D ALCT: For binary decomposition as shown in Fig 2, if we fix the window width $|I|$; all wavelets in Eq.(4) or Eq.(5) can form an orthonormal basis for $L^2(R)$ and this basis is called level 1; if we then select $|I|/2$ as the window width, the wavelets in Eq.(4) or Eq.(5) can also form an orthonormal basis and this is called level 2; and so on. Thus, a binary-based signal decomposition tree consists of the bases at different levels. However, not all the bases are efficient in matching a given signal. Therefore, we must pick the best-basis from all the possible local cosine/sine bases, using a cost-functional. To search for the local cosine/sine best-basis, i.e. an adaptive local cosine/sine basis, in order to achieve the best matching to the signal, a cost-functional is defined based on the entropy of the decomposed signal. There are numerous cost functional that can be used. Here, we use Shannon entropy as the cost-functional [3]], i.e.

$$E(X) = -\Sigma_n p_n log_2(p_n) \tag{11}$$

where $p_n = |x_n|^2/||X||^2$; X is the signal of length N samples, x_n is the nth component of X and $E(X)$ denotes the entropy of the signal X. In implementation, we use Coifman

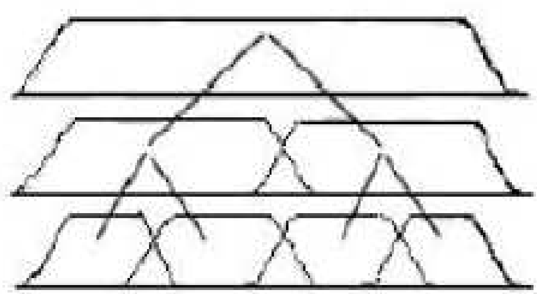

Fig. 2: Binary tree decomposition[[2]]

and Wickerhauser's fast algorithm to search for the best-basis based on the Shannon entropy. The main concept of this algorithm is that the full local cosine/sine tree is pruned recursively at each node by comparing its entropy to the summation of the entropy of its corresponding child nodes, i.e.

if Entropy(parent node) $\leq$ [Entropy(child1) + Entropy(child2)] then cut off the child branches:

Initially, a full binary-based decomposition tree with a preset maximum decomposition level is produced. The pruning procedure then starts from the leaf nodes and proceeds towards the root. At the end of this procedure, an optimal pruned tree is obtained for the given signal, i.e. an adaptive local cosine/sine basis is obtained.

6. Adaptive Tiling of Images

As explained in section 5, we can adaptively select the size and location of the windows with the best basis algorithm. We consider only tiling that can be generated from separable bases. We divide the image into four sub squares, and we consider the local cosine basis associated with this tiling. We

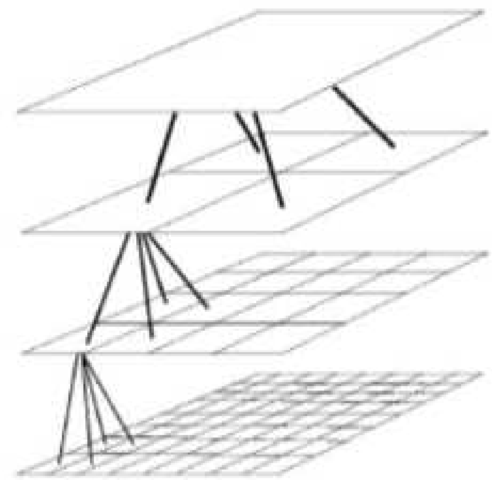

Fig. 3: Quad-tree decomposition of the Image [[2]]

then further decompose each square into four sub squares, and consider the local cosine bases associated with this finer tiling. By applying this decomposition recursively we obtain a homogeneous quad tree-structured decomposition as shown in Fig.3. As in the 1-D, for each sub block, or node of the quad tree, we calculate the set of coefficients in the sub block. We associate a cost for each node of the tree, based on the set of coefficients, and we find an optimal segmentation of image.

7. Experimental Results

The ALCT method is applied on a standard 2-D marine seismic data of size 512 x 512 samples and its performance is evaluated on the basis of compression Ratio (CR) and SNR as tabulated in Table.1. The performance is compared with compression using fast bi-orthogonal wavelets along with Embedded zero tree coding and adaptive arithmetic coding method[[1]].The original and reconstructed images are as shown in Fig.4 and Fig.5.

Table 1: Comparison of SNR of the ALCT method with Fast wavelet method [[1]] for different CR

Compression Ratio	ALCT SNR(db)	Fast Wavelet with EZW coding SNR(db)
10	42.6	40.7
20	42.1	40.02
30	33.7	33.4
40	21.08	26.24
50	14.04	17.07

8. Conclusion

Compression algorithms, which work well for multimedia images, such as EZW and SPIHT compression schemes, perform poorly on seismic data. However the method presented in this paper based on Adaptive Local cosine transform are well suited to capture oscillatory nature of the image. It is also seen that these algorithms work well on the actual seismic data giving good compression ratios and SNR. From the experiments conducted it is seen that presented seismic data compression algorithms, at moderate compression ratios of 10:1 to 20:1 are well-suited and safe for seismic data processing and interpretation.

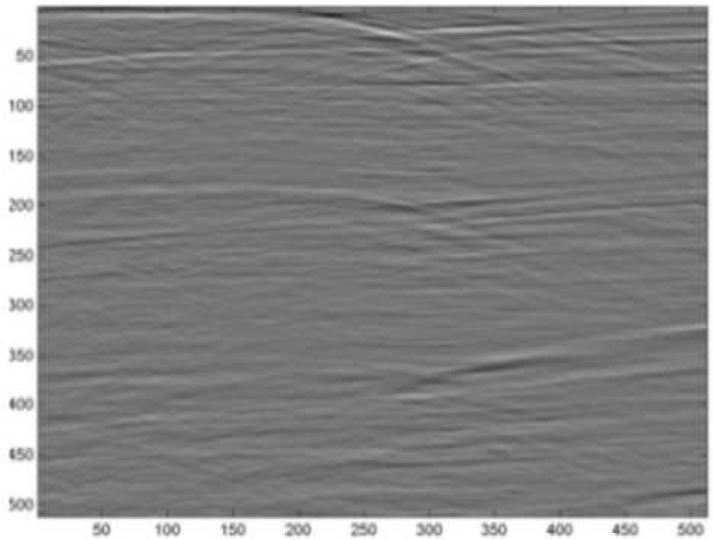

Fig. 4: The original seismic Image.

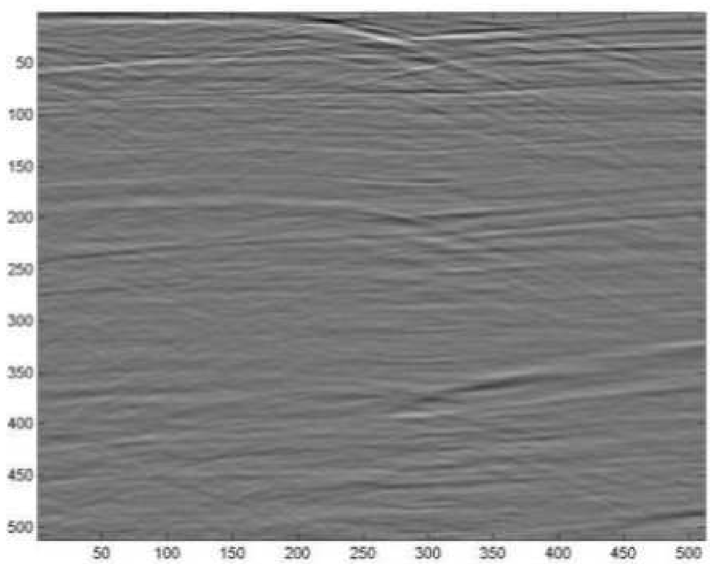

Fig. 5: The reconstructed image using ALCT with a compression ratio of 21.1 and SNR of 40.02 db

REFERENCES

[1] Aparna.P ,"Seismic Data Compression using Fast Wavelets and Local Cosine Transform",*M.Tech Thesis*, 2004.

[2] Amir Z. Averbuch, F. Meyer, J.O. Strmberg, R.Coifman, and A. Vassiliou, " Low Bit-Rate Efficient Compression for Seismic Data ", *IEEE Transactions on Image processing*, Vol. 10, No.12, December 2001, pp. 1801-1814.

[3] Yongzhong Wang and Ru-Shan Wu," Seismic data compression by an adaptive local cosine/sine transform and its effects on migration", *Geophysical Prospecting*, Vol 48,Number 6, November 2000, pp. 1009-1031.

[4] G. Aharoni , A. Averbuch , R. Coifman , M.Israeli ,"Local cosine transform a method for the reduction of the Blocking Effect in JPEG.",*Journal of Mathematical Imaging and Vision, Special Issue on Wavelets*, Vol. 3, 1993, pp. 738.

[5] R. R. Coifman and M. V. Wickerhauser, "Entropy-based algorithms for best basis selection.", *IEEE Trans. Inform. Theory*, Vol.38,Mar.1992, pp. 713718.